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Massive Scalar Field Kaluza-Klein Cosmological Model in Lyra's Manifold

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Abstract: **Five dimensional Kaluza-Klein cosmological model is investigated in the framework of Lyra'smanifold in the presence of massive scalar field.Exact solutions to the field equations are derived. Some physical & kinematical properties of the modelare also discussed.**

Keywords**: Kaluza–Klein, Lyra's geometry, massive scalar-fields.**

I. INTRODUCTION

Kaluza(1921)and Klein (1926) have proposed a hypothesis to bind together the fundamental forces in nature to be specific gravitation and electromagnetism. This hypothesis from a specific perspective takes after the ordinary gravity in free space aside from that it is expressed in five dimension instead of four dimension. This hypothesis prompts a fascinating chance known as "cosmological reduction process" in which the size of additional dimensions becomes so small as to be imperceptible by experiencing contraction. Chodos and Detweller (1980) and Freund and Hawking examined such cosmological models and have demonstrated that the additional dimensions are inconspicuous because of dynamical contraction. Appelquist*et al.* (1987) has additionally explored some cosmological models in five dimensional Kaluza-Klein space-time. Kaluza-Klein cosmological models have been examined by a few creators inmodified theories of gravity are Samanta*et.al.*(2016), Aditya *et.al*.(2019),Naidu *et.al* (2021), Reddy *et.al*.(2019).

In recent, years there has been a massive interest in the examination of scalar fields as a result of the way that they assume a significant part in cosmology. Genuinely the represent matter fields with spin less quanta and portray gravitational field in free space. It is additionally notable that they cause accelerated expansion of the universe. Scalar fields can be characterized into two classes. They are mass less (zero mass) scalar fields and massive scalar fields. Massless scalar fields gives long range interactions and massive scalar fields portray short range interactions. Here we are interested inthe zero mass scalar fields.Aygun*et.al* (2012) investigated the massive scalar field for a Marder type universe in the framework of Lyra and Riemannian geometries. Also, detailed discussion of massless scalar field is contained in the work of Venkateswarlu (2015), Staykov*et al.* (2018).

Lyra (1951) proposed an alteration of Riemannian geometry by introducing gauge function into the structure less manifold, because of which the cosmological constant emerges naturally from the geometry. This bears a wonderful similarity to Weyl's (1918) geometry. In resulting examinations Sen (1957) and Sen and Dunn (1971) formed another scalar-tensor theory of gravitation and built an analogof the Einstein field equations depend on Lyra's geometry. Halford (1972) has demonstrated that the scalar-tensor treatment depend on Lyra's geometry predicts similar impacts as in generalrelativity.KalyaniDesikan (2020) has examined cosmological models in Lyra geometry with time varying displacement field. Also D C Maurya*et.al.* (2019), D.D.Pawar*et. al.* (2020) havediscussed cosmological model in Lyra geometry and massless scalar field.

The purpose of this paper is to investigate the role of a massive scalar field distribution for Lyra's geometries with the aid of Kaluza-Klein space time.

II. THE METRIC AND FIELD EQUATIONS

We consider five- dimensional Kaluza-Klein metric in the form,

$$ds^2 = dt^2 - A^2(t)(dx^2 + dy^2 + dz^2) - B^2(t)d\varphi^2 \quad (1)$$

Where, A and B are functions of cosmic time t and fifth coordinate φ is space-like.

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The relativistic field equations in normal gauge in Lyra's manifold are as,

$$R_{ij}-\frac{1}{2}g_{ij}R+\frac{3}{2}\phi_i\phi_j-g_{ij}\phi_k\phi^k=-T_{ij} \quad (2)$$

Where ϕ is a displacement vector field and the other symbols have their usual meaning as in Riemannian geometry. We now assume the vector displacement field β to be the time like constant vector.

$$\phi_i=\left(\beta(t),0,0,0,0\right) \quad (3)$$

$\beta=\beta(t)$ is a function of time alone.

The energy momentum tensor for massive scalar field is given by,

$$T_{ij}=V_{,i}V_{,j}-\frac{1}{2}g_{ij}\left(V_{,k}V^{,k}-M^2V^2\right) \quad (4)$$

Where V is the massive scalar field and a function of t. M is the mass of a zero-spin particle ,the scalar field V satisfies the following Klein-Gordon wave equation:

$$g^{ij}V_{;ij}+M^2V=0 \quad (5)$$

The field equations (2) and (3) for the metric (1), with the help of (4) can explicitly be written as

$$3\left[\left(\frac{\dot{A}}{A}\right)^2+\frac{\dot{A}\dot{B}}{AB}\right]=\frac{\dot{V}^2}{2}+\frac{M^2V^2}{2}+\frac{3}{4}\beta^2 \quad (6)$$

$$2\frac{\ddot{A}}{A}+\left(\frac{\dot{A}}{A}\right)^2+2\frac{\dot{A}\dot{B}}{AB}+\frac{\ddot{B}}{B}=-\frac{\dot{V}^2}{2}+\frac{M^2V^2}{2}-\frac{3}{4}\beta^2 \quad (7)$$

$$3\left[\frac{\ddot{A}}{A}+\left(\frac{\dot{A}}{A}\right)^2\right]=-\frac{\dot{V}^2}{2}+\frac{M^2V^2}{2}-\frac{3}{4}\beta^2 \quad (8)$$

From equation (5), we have

$$\ddot{V}+\dot{V}\left(3\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)+M^2V=0 \quad (9)$$

III. SOLUTIONS OF THE FIELD EQUATIONS

Here, we have three independent field equations (6) to (8) connecting four unknownparameters A,B,β,V and therefore in order to obtain exact solution. We assume a functional relationship between the metric coefficient A and B of the form,

$$A=B^n \quad (10)$$

where 'n' is an arbitrary constant.

Using this relation, the field equations (6) to (8) admit the exact solution and we obtain metric coefficient as

$$A=(n+3)^{\frac{1}{n+3}}\left(k_1t+k_2\right)^{\frac{1}{n+3}} \quad (11)$$

$$B=(n+3)^{\frac{n}{n+3}}\left(k_1t+k_2\right)^{\frac{n}{n+3}} \quad (12)$$

Usingequation (11) and (12) in equations (6) to (8), we obtain two equations as follows

$$\frac{3k_1^2(n+1)}{(n+3)^2(k_1t+k_2)^2}=\frac{\dot{V}^2}{2}+\frac{M^2V^2}{2}+\frac{3}{4}\beta^2 \quad (13)$$

$$\frac{-3k_1^2(n+1)}{(n+3)^2(k_1t+k_2)^2}=\frac{-\dot{V}^2}{2}+\frac{M^2V^2}{2}-\frac{3}{4}\beta^2 \quad (14)$$

From equations (13) and (14) we obtain

$$M^2V^2=0 \quad (15)$$

Then there are arises two cases

(i)Massless scalar field and

(ii)Vacuum solutions

(i) Massless scalar field : In this case, we have from equation (15) obtain $M=0$ using in the Klein-Gordon equation (9), the scalar field function is obtain as follows:

$$V=\frac{k_3}{(n+3)}\log\left(k_1t+k_2\right)+k_4 \quad (16)$$

Where k_3 and k_4 are arbitrary integration constants. Also, using equations (11),(12) and $M=0$ in field equations (6) to (8),the displacement field β^2 function is obtain as follows:

$$\beta^2=\frac{4}{3}\left[\frac{3k_1^2(n+1)-k_3^2}{(n+3)^2(k_1t+k_2)^2}\right] \quad (17)$$

(ii) Vacuum solutions:

In this case, we have $V=0$ from equation (15) , using in the Klein-Gordon equation (9), the scalar field function is vanish and we obtain the vacuum solution for the kaluza-klein universe in Lyra geometry. In the vacuum solution , β^2_{vacuum} function is obtain as follows:

$$\beta^2_{vacuum}=\frac{4}{3}\left[\frac{3k^2_1(n+1)}{(n+3)^2(k_1t+k_2)^2}\right] \quad (18)$$

Now using equation (11) and (12) five-dimensional kaluza-klein cosmological model can be written as;

$$ds^2=dt^2-P^2(k_1t+k_2)^{\frac{2}{n+3}}(dx^2+dy^2+dz^2)-R^2(k_1t+k_2)^{\frac{2n}{n+3}}d\varphi^2$$

Where, $P=(n+3)^{\frac{1}{n+3}}$, $R=P^n$

This model can be transformed through a proper choice of coordinates to the form

$$ds^2=\frac{dT^2}{k^2_1}-P^2T^{\frac{2}{n+3}}\left(dx^2+dy^2+dz^2\right)-R^2T^{\frac{2n}{n+3}}d\varphi^2 \quad (19)$$

Where $T=k_1t+k_2$

IV.THE PHYSICAL AND KINEMATICAL PROPERTIES

In this section, we discuss some physical and kinematical properties of the Kaluza-Klein model (19).

SpatialVolume : $V=RP^3T$ (20)

Expansion Scalar : $\theta=3\frac{\dot{A}}{A}+\frac{\dot{B}}{B}=\frac{k_1}{T}$ (21)

Hubble Parameter: $H=\frac{1}{4}\left(3\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)=\frac{k_1}{4T}$ (22)

Shear Scalar : $\sigma^2=\frac{k^2_1n(n-3)}{3(n+3)^2T^2}=\frac{k_3}{T^2}$ (23)

Vorticity : $\omega_{11}=\omega_{22}=\omega_{33}=\omega_{44}=0$ (24)

It may be observed that initial moment ,when $T\to 0$ the spatial volume will be zero and scalar expansion , shear scalar and Hubble parameter tends to ∞. For large values of T , we observe that scalar expansion ,shear scalar, Hubble parameter becomes zero.Here $\left(\frac{\sigma^2}{\theta^2}\right)$ does not vanish for a large value of T which implies that the model (19) anisotropic and does not approach isotropy. Vorticity of model along x,y,z and t is zero. So, the obtained model is non-rotating.From equations (17) and (18) , the β^2 and β^2_{vacuum} functions decrease with time and from equations (16) and (17) obtain the relation between the displacement field β^2 and the scalar field V as follows

$$\beta^2=\frac{4}{3}\left[\frac{3k^2_1(n+1)-k^2_3}{(n+3)^2e^{2(v-k_4)(n+3)/k_3}}\right] \quad (25)$$

Graphically, variation of displacement field β^2 and the scalar field V are as follows.

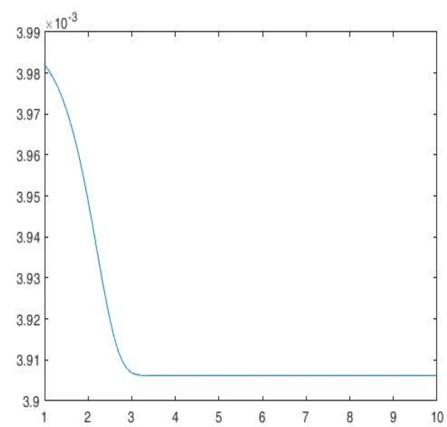


Fig.1:variations of beta in lyra geometry

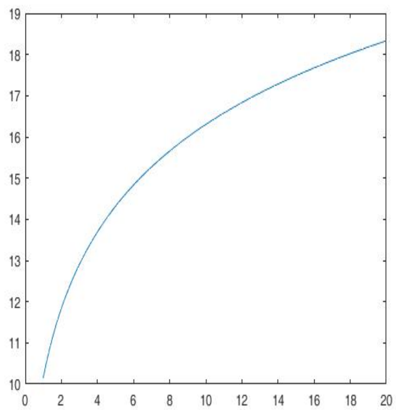


Fig.2: variations of v in lyra geometry

From the figure there is an inverse relation between displacement field β^2 and scalar field V.

The graphical representation of Spatial volume, Hubble parameter, Expansion Scalar, Shear Scalar are as follows

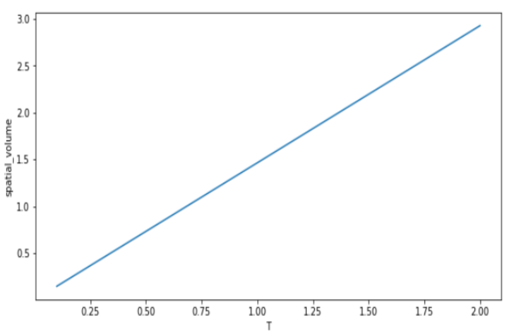
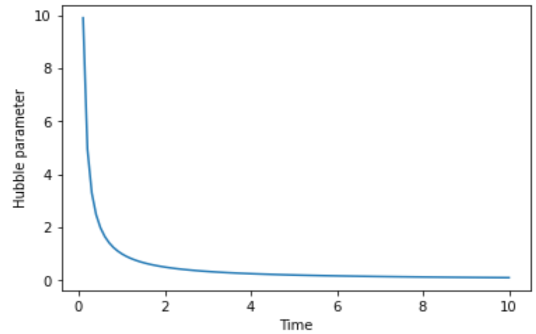
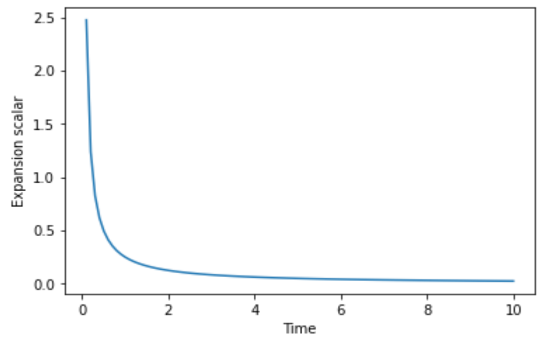


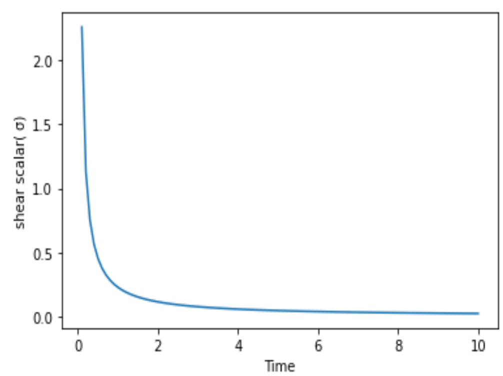
Fig.3: Spatial volume vs Time



(fig.:4) Hubble parameter vs Time



(fig. 5) expansion scaler vs Time



(fig. 6) shear scaler vs Time

CONCLUSION

In this paper, we found exact solutionsof thefive dimensional Kaluza-Klein space time with a massive scalar field distribution in presence of Lyra's Manifold. Here, we have investigated two cases i.e. a massless scalar field and a vacuum solution. From equation (15) it is obvious that, a massive scalar field does not exist in the Lyra manifold.

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REFERENCES

[1]. Aditya, Y.Deniel,K. , Rao VUM., Reddy DRK. (2019).Kaluza-Klein dark-energy Model in Lyra Manifold in the presence of massive scalar field. Astrophysics Space Sci. 364(11),1-8.

[2].Appelquist, T, Chodos, A, Freund PGO. (1987). Modern Kaluza-Klein Theories (Frontiers in Physics). Addison-Wesley.

[3]. Aygün, S., Aktaş, C., Yılmaz, I. (2012). Non-existence of a massive scalar field for the Marder universe in Lyra and Riemannian geometries. Journal of Geometry and Physics, 62(1),100–106.

[4]. Chods, A. and Detweller, S. (1980). Where has the fifth dimension gone? Phys. Rev. D21,2167-2170.

[5]. Desikan, K. (2020). Cosmological models in Lyra geometry with time varying displacement field. The African review of physics.

[6].Freund. PGO, Hawking SW. The Pontifical Academy of Sciences

[7]. Halford, W.D. (1972). Scalar-Tensor Theory of Gravitation in a Lyra Manifold. Journal of Mathematical Physics 13:1699.

[8].Kaluza,T.(1921).ZumUnitatsproblem. der Physik,Sitzungber.presuss, Akad , wissBertin (maths.Phys),966-972.

[9].Klein, O. (1926). Quantum Theory and Five-Dimensional Theory of Relativity. Z. Phys.37(12), 895-906.

[10] Lyra, G. (1951). Uber eine modification der Riemannschen Geometric, Math Z 54:52-64.

[11] Maurya, D.C, Rashid Zia (2019). Brans-Dicke Scalar Field Cosmological model in Lyras Geometry. Physical review D –APS

[12] Naidu. K.D., Aditya. Y., Naidu, R.L., Reddy, DRK. (2021).Kaluza-Klein Minimally interacting dark energy Model in the presence of Massive scalar field. Modern Physics letter A ,Vol.36(08) 2150054.

[13] Pawar, D.D., Dagwal, V.J., Solanke,Y..S (2020) .Titled universe with big rip singularity in Lyra geometry. Modern physics letter A vol.35(24) 2050196.

[14].Reddy,DRK.,Ramesh,G.(2019).Five-Dimensional anisotropic Dark energy Cosmological Model in the presence of scalar-Meson fields in General Relativity,Vol-1(2),60-70.

[15].Samanta, G.C. , BishiBinaya (2016). Universe Described by Kaluza-Klein Space Time with Viscous Modified cosmic Chaplygin Gas in General Relativity. Iranian Journal of Science and Technology,Transaction A. Science 40(4),245-254.

[16].Sen, D.K.(1957) .A Static Cosmological Model. Z Physik 149,311-323.

[17] Sen, D.K. and Dunn, K.A.(1971).A scalar-Tensor Theory of Gravitation in a Modified Riemannian Manifold. Journal of Mathematical Physics 12: 578.

[18] Staykov, K.V., Popchev. P., Doneva, D.D. (2018). Static and slowly rotating neutron stars inscalar-tensor theory with self-interacting massive scalar field. The European physical journal C 78 :586.

[19] Venkateshwarlu, R., Satish,J. (2015). Bulk viscous cosmic strings coupled with mass less scalar field. Journal of Advanced physics, vol.4 (2): 101 -112.

[20] Weyl,H.(1918).Infiniteimal geometric, Math Z2:384-411.

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